1. (Lewand, Exercises 1.1, #7) Prove that $n < 2^n$ for all integers $n \geq 1$.

2. (Lewand, Exercises 2.2, #2) Evaluate $C(7, 5)$ and $C(7, 2)$. Explain why you get the same answer for both expressions. Generalize your discovery to $C(n, r)$ and $C(n, n-r)$.

3. (Lewand, Exercises 2.2, #3) Evaluate $P(7, 5)$ and $P(7, 2)$. Explain why you do not get the same answer for both expressions.

4. (Lewand, Exercises 2.3, #3) Four cards are to be drawn simultaneously from a standard deck of 52. Find the probability that:
   
   (a) all four are clubs.
   
   (b) at least one is not a club.
   
   (c) three are black and one is red.
   
   (d) exactly two are picture cards.
   
   (e) either two or three of the four cards are picture cards.

5. (Lewand, Exercises 2.3, #8) A lottery consists of 48 numbers, of which 8 are selected by the lottery authority. To win, you need to have played either 7 or 8 of the numbers selected by the authority. If you pick exactly 7, you win $20,000; if you pick 8 you win $50,000. What is the probability that you
   
   (a) win $50,000?
   
   (b) win $20,000?

6. Write a program that takes a keyword as a command line argument, then performs the Vigenère cipher on the standard input.

7. (Extra credit for undergrads, required of graduate students) Modify your program to decrypt the Vigenère cipher, again reading the key from standard input. (You could just add a -d flag to your program.) This is not a program to cryptanalyze an unknown Vigenère cipher.