My signature testifies that I have neither given nor received aid on this quiz.

Name:________________________ SUID:____________________

CIS 400/628
Spring 2005
Quiz 2

1. (3 pts.) For a given moduls m, the multiplicative inverse of a number a is the number b such that $ab = 1$ (mod m). Find the following multiplicative inverses (note that no even number can be a multiplicative inverse for an even modulus):

   (a) 3 (mod 4)
   $3 \times 3 = 9$ (mod 4) $= 1$, so 3^{-1} (mod 4) $= 3$.

   (b) 3 (mod 5)
   $3 \times 2 = 6$ (mod 5) $= 1$, so 3^{-1} (mod 5) $= 2$.

   (c) 5 (mod 26)
   $5 \times 21 = 105$ (mod 26) $= 1$, so 5^{-1} (mod 26) $= 21$.

2. (2 pts.) A standard deck of playing cards has four suits (clubs, diamonds, hearts, and spades), of which two (diamonds and hearts) are red, and two (clubs and spades) are black. Each suit has 13 cards, namely the Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, and King. The Jack, Queen, and King are called *picture* or *face* cards (because they have pictures of people's faces on them). Given a standard 52-card deck that is randomly shuffled between choices of a card, then:

   (a) If an experiment consists of choosing a card from a shuffled deck, and I repeat this experiment 1000 times, what is the expected number of Aces or hearts?
   P(A) = 4/52 = 1/13.
   P(H) = 13/52 = 1/4.
   P(A ∩ H) = 1/52.
   P(A ∪ H) = 4/52 + 13/52 - 1/52 = 16/52 = 4/13.
   (conceptually: 13 hearts + 3 other aces = 16 cards / 52 total)
   1000 × 4/13 = 307.7. (about 308 such cards expected)

   (b) A card is selected at random, and then re-inserted into the deck; the deck is reshuffled, and another card is chosen. Let E be the event "the first card is a Jack" and let F be the event "the second card is black". Are E and F independent events? Compute $p(E \cap F)$.
   P(1J) = 4/52 = 1/13.
   P(2black) = 1/2.
   Yes, they're independent (because the card is replaced).
   P(1J ∩ 2black) = 1/13 × 1/2 = 1/26 = 0.0385 (3.85%)

3. (5 pts.) I have 3 vests (red, tan, yellow), 10 shirts that my wife will allow me to wear to work, and 5 pairs of pants.

   (a) How many different choices do I have for my base outfit? Note that I don't *always* wear a vest, but, except in nightmares, I always wear a shirt and pants to work.
   4 vests $\times$ 10 shirts $\times$ 5 pants = 200.
   Vests are 4 because there's an additional "no vest" option beyond the three colors.

   (b) Some days days I accessorize with one or both of my USB flash memory sticks on a lanyard; some days I go without. I also have a pair of sunglasses that protect my eyes from the yellow face, which I might or might not remember to wear. Counting these accessories, how many outfits can I make now?
   200 $\times$ 4 (no USB, silver, black, both) $\times$ 2 (sunglasses, no) = 1600

   (c) This being Syracuse, it is only 25% likely that I will need my sunglasses, and it's 60will wear *a* vest (Note that my yellow vest, garish as it is, is my favorite, as my younger son picked it out for me; I'll choose it 50% of the time I choose a vest. The other two are equally likely if I choose a vest but don't choose the yellow one). On any given day, I'm 50% likely to wear my silver USB stick, and also 50% likely to wear my black USB stick. What's the probability that I come to work with:

      i. My sunglasses and any vest?
      .25 (sunglasses) $\times$ .6 (some vest) = .15 (15%)

      ii. My yellow vest but no sunglasses?
      [.6 (some vest) $\times$ .5 (yellow vest)] $\times$ 0.75 (no sunglasses) = .3 $\times$ .75 = .225 (22.5%)

      iii. My red vest, sunglasses, and no USB stick?
      [.6 (some vest) $\times$ .25 (red vest)] $\times$ .25 (sunglasses) $\times$ .25 (no USB stick) = .009375 (0.94%)
      (Note: P(No USB stick) = .5 (no silver) $\times$ .5 (no black) = .25)