The question was raised in class, if $gcd(a,b) = d$, then for any positive integer k, does $gcd(ka,kb) = kd$?

Recall from the fundamental theorem of arithmetic that any positive integer can be factored into primes with a unique representation, that is, for any positive integer x,

$$x = p_1^{m_1} \cdot p_2^{m_2} \cdot p_3^{m_3} \cdot \ldots \cdot p_j^{m_j}$$

This can equivalently be written:

$$x = 2^{m_2} \cdot 3^{m_3} \cdot 5^{m_5} \cdot \ldots \cdot p_j^{m_j}$$

if we allow some of the m_i to be 0. In particular, all of a, b, and d can be represented thus:

$$a = 2^{m_2} \cdot 3^{m_3} \cdot 5^{m_5} \cdot \ldots \cdot p_j^{m_{p_j}}$$
$$b = 2^{n_2} \cdot 3^{n_3} \cdot 5^{n_5} \cdot \ldots \cdot p_j^{n_{p_j}}$$
$$d = 2^{l_2} \cdot 3^{l_3} \cdot 5^{l_5} \cdot \ldots \cdot p_j^{l_{p_j}}$$

In this representation, each $l_i = min(m_i, n_i)$. That is, the exponent for each prime in the factorization of d is the minimum of the two exponents in the factorizations of a and b:

$$d = 2^{min(m_2,n_2)} \cdot 3^{min(m_3,n_3)} \cdot 5^{min(m_5,n_5)} \cdot \ldots p_j^{min(m_{p_j},n_{p_j})}$$

Now, k can also be factored the same way, i.e.,

$$k = 2^{o_2} \cdot 3^{o_3} \cdot 5^{o^5} \cdot \ldots \cdot p_j^{o_{p_j}}$$

$$\therefore ka = 2^{m_2+o_2} \cdot 3^{m_3+o_3} \cdot 5^{m_5+o_5} \cdot \ldots \cdot p_j^{m_{p_j}+o_{p_j}}$$
$$kb = 2^{n_2+o_2} \cdot 3^{n_3+o_3} \cdot 5^{n_5+o_5} \cdot \ldots \cdot p_j^{n_{p_j}+o_j}$$

and

$$= gcd(ka,kb) = 2^{min(m_2+o_2,n_2+o_2)} \cdot 3^{min(m_3+o_3,n_3+o_3)} \cdot 5^{min(m_5+o_5,n_5+o_5)} \cdot \ldots \cdot 2^{min(m_{p_j}+o_{p_j},n_{p_j}+o_{p_j})}$$
$$= 2^{o_2} \cdot 3^{o_3} \cdot 5^{o^5} \cdot \ldots \cdot p_j^{o_{p_j}} \cdot 2^{min(m_2,n_2)} \cdot 3^{min(m_3,n_3)} \cdot 5^{min(m_5,n_5)} \cdot \ldots p_j^{min(m_{p_j},n_{p_j})}$$
$$= 2^{o_2} \cdot 3^{o_3} \cdot 5^{o^5} \cdot \ldots \cdot p_j^{o_{p_j}} \cdot \mathrm{gcd}(a,b)$$
$$= k \cdot \mathrm{gcd}(a,b) \quad \square$$